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DISCOVERY

The accounts of Gompertzian mortality MODEL: Evidence from indirect analytical approach

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ABSTRACT

This paper aims to carry out a thorough examination of Gompertzian mortality models based on English population table, specifically the, objectives are to estimate the parameters in Gompertz model and use the estimated parameters to construct mortality tables. In order to achieve these objectives, we used simple logarithmic transformation to estimate the parameters of Gompertz model and obtained models for both survival function and the force of mortality using expected number of survivors from population of England. The model derived is then used to generate mortality table based on Gompertz law. The major advantage of the Gompertz model is its simple determination of the whole life table with all its demographical and actuarial parameters. Unlike other models, the Gompertz model is not restricted to the forecast of death probability but also the forecast of all life table parameters.

Key words: mortality, Gompertz, logarithmic, parameters, survivors

1. INTRODUCTION

The resultant effects of not adopting an adequate mortality model reflecting survival patterns of potential and existing clients can create underwriting upset. If the life office overestimates its risks, then the insured will bear the financial burden of excessive insurance premiums which may probably call for regulatory inquiries. Mortality analysis has been a major concern in life insurance and since human lifespan has increased tremendously, actuaries are now keen interest in mortality rate and the prediction of future life uncertainties. In Booth & Tickle (2008), it was reported that the continuous increment in life expectancy beyond initial held bounds has led to the criticality of mortality forecasting. While estimating the force of mortality numerically in Ogungbenle & Adeyele (2020), it was observed that the distribution of deaths comprises of decline in the high number of deaths with age after birth to account for infant mortality and deaths centered around the late modal age, accounting for senescent mortality and premature deaths occurring infrequently at tenderly ages between high infant mortality and senescent deaths. In Missov, Lenart, Nemeth, Canudas-Romo, Vaupel (2015), the Gompertz force of mortality was derived as functions of the initial level of mortality and the rate at which mortality increases with age. The authors obtained the Gompertz force of mortality as functions of band, the old age, modal age at death M(x) and expressed similar relationships for other popularly applied mortality models so as to explain the merits of applying the parameters in terms of M(x).

Gompertz model expresses mathematically the age schedule of mortality, that is, mortality as a function of age in a given year. However, in Cohen (2018), Mortality models are different in the number of parameters used and in the age band for which they model mortality well. The more parameters used, the more flexible mortality can be fit at different ages, but the more difficult they are to analyze actuarially.

Mortality table is a good summary tool for appraising and comparing mortality conditions prevalent in populations. Its construction therefore needs reliable mortality data in a population's mortality rates by age and sex. It was reported in Van der Mulen (2012) that the variations in the types of life tables are dependent on the peculiarity of mortality data used for construction. The most reliable source of such data is a functional vital statistics where deaths and births are registered. Deaths at each age are related to the size of the population in that age band usually which is estimated from population censuses or continuous registration of all births and deaths and consequently the resulting age-sex-specific death rates are then used in computing life table.

Following Li & Tuljapurkar (2014), the starting number of the hypothetical cohort, l_o is arbitrary in deterministic models and is usually taken as 100,000. Apart from insurance underwriting information and the existing analytic mortality models, Mortality forecasts are based on these tables making mortality models to be of equal importance. In Bowers, Newton L, Gerba H.U,. Hickman J.C, Jones D.A and Nesbitt C.J.(1997),it was observed that mortality models are described in analytic form, because of philosophical, practical and ease of computation. In view of the authors, the philosophical justification ensures that, since a few physical concepts are explained efficiently by actuarial model, using biological process, human survival can be modeled by an equally easy actuarial function which is numerically convenient to estimate and interpret for mortality purposes. Furthermore, for practical reasons, applying the actuarial phenomena of model parsimony makes it convenient to describe a mortality function with some parameters than to describe a life table with many parameters. However, an analytic and smooth survival function should have few parameters so as to enable estimation from mortality data. Because human populations have resulted in higher longevity, the Gompertzian slope has steadily increased indicating a more or less severe relationship between ageing and increasing mortality at different ages Tai (2017). However, the baseline mortality rate at different ages has dropped to account for the concomitant increases in the gradient. This underscores the multidimensionality of population average longevity.

2. PROCEDURE OF MORTALITY INTENSITY

q_x define the proportion of life exactly aged x who die before attaining age $x+1$. Mortality table is computed differently for both male and female because of the differences in mortality of both male and female.

μ_x : In actuarial mathematics, the intensity of mortality describes the instantaneous rate of mortality at a certain age measured on an annual basis.

l_0 : Entry population in a study known as radix.

$l_{x+0} = \int_0^\infty l_{x+s}\mu_{s+s}ds$ = Expected number of survivors attaining age x

l_{x+1} = Expected number of survivors attaining age $x+1$

dx =The number of lives dying between age x and $x+1$

$$d_x = \int_0^\infty l_{x+s}\mu_{s+s}ds - \int_1^\infty l_{x+s}\mu_{s+s}ds$$

T_x = Complete future lifetime of life age x

L_x = Number of persons years lived age x

$\overset{\circ}{e}_x$ = Expected future life time measuring expected time remaining until death.

The curtate future life time of a life age x representing integral part of T_x

The force of mortality, $\mu_x(s)$, as an instantaneous rate mortality at age x in year s is defined

$$\mu_x = \lim_{\delta x \to 0} \frac{\{x < T_{0(s-x)} \le x + \delta x \mid T_{0(s-x)} > x\}}{\delta x} \quad (1)$$

Where $T_{0(s-x)}$ is the outstanding life time of an individual born at age $s-x$. This individual will die at age x + $T_{0(s-x)}$ in year $s+T_{0(s-x)}$. l_x is the number of lives surviving in a closed group, attaining age x. Considering the continuously smooth nature of the curve, μ_x is defined on this curve by the gradient of l_x function which serves as ratio of the instantaneous rate of decrease of l_x to the corresponding value of l_x

$$\mu_x = -\frac{1}{\int_0^\infty l_{x+s}\mu_{s+s}ds}\frac{d}{dx}\int_0^\infty l_{x+s}\mu_{s+s}ds \quad (2)$$

3. MATERIAL AND METHODOLOGY

Gompertz first observed that a law of geometric progression, after certain age, in many populations and modeled the mortality risk as:

$$\mu_X = ae^{bx} \quad (3)$$

The mortality hazard rate can be written as the sum of a constant term BC^X as capturing the hazard of aging. Where a, b, are constants and $e = 2.71$

Therefore $l_x = \kappa\lambda^{\zeta^x}$ (4)

Where κ, λ and ζ are parameters; $\kappa = e^{c_1}$, c_1 is an integrating constant, $\lambda = e^{-\frac{a}{b}}$ and $\zeta = \exp b$

Given that $l_{20} = 98496, l_{30} = 97645, l_{40} = 96500$ from English life table 15 male, we can estimate the values of the parameters in our model

$$z = \log_e l_x \quad (5)$$

Therefore $z = \log_e \kappa + \zeta^x \log_e \lambda$ (6)

$$z_1 = A + B\zeta^{x_1}, z_2 = A + B\zeta^{x_2}, z_3 = A + B\zeta^{x_3}$$

$$z_2 - z_1 = A + B\zeta^{x_2} - A - B\zeta^{x_1} \quad (7)$$

$$z_2 - z_1 = B(\zeta^{x_2} - \zeta^{x_1}) \quad (8)$$

$$\frac{z_3 - z_2}{z_2 - z_1} = \frac{B(\zeta^{x_3} - \zeta^{x_2})}{B(\zeta^{x_2} - \zeta^{x_1})} = \zeta^{\delta}, \zeta = \left(\frac{z_3 - z_2}{z_2 - z_1}\right)^{\frac{1}{\delta}} \quad (9)$$

$$\lambda = \exp\left[\frac{(z_2 - z_1)^2}{z_3 - 2z_2 + z_1}\left(\frac{z_2 - z_1}{z_3 - z_2}\right)^{\frac{x_1}{\delta}}\right], \kappa = \exp\left[z_1 - \frac{(z_2 - z_1)^2}{z_3 - 2z_2 + z_1}\right] \quad (10)$$

$$x_1 = 20, x_2 = 30, x_3 = 40$$

$$z_1 = \log_e l_{x_1} \Rightarrow \log 98496; z_2 = \log_e l_{x_2} \Rightarrow \log 97645; z_3 = \log_e l_{x_3} \Rightarrow \log 96500$$

$$\delta = x_2 - x_1, x_3 - x_2 \Rightarrow 30 - 20, 40 - 30, = 10; x_1 = 20, x_2 = 30, x_3 = 40$$

$$\zeta_{male} = 1.031174114, \lambda_{male} = 0.987014938, and\,\kappa_{male} = 100903.6449$$

$$\mu_x = ae^{bx}, \zeta = e^b; therefore \log_e \zeta = b, \log_e \zeta = 0.030698069$$

$$b = 0.030698069; \lambda = e^{-\frac{a}{b}} \Rightarrow \log_e \lambda = -\frac{a}{b}$$

$$b\log_e \lambda = -a, -b\log_e \lambda = a; \log_e \lambda = -0.013070105$$

$$a = -0.030698069 \times -0.013070105; a = 0.000401227$$

Therefore $\mu_x = 0.000401227e^{0.030698069x}$

$$l_x = 100903.6449 \times 0.987014938^{1.031174114^x}$$

From English life table 15Female $l_{20} = 98957, l_{30} = 98617, l_{40} = 97952$

$$\zeta_{female} = 1.069931341, \lambda_{female} = 0.999078419, \kappa_{female} = 99310.24193$$

Therefore $\mu_x = ae^{bx}, \zeta = e^b \Rightarrow \log_e \zeta = b; l_x = 99310.24193(0.999078419)^{1.069931341^x}$

$$b = \log_e 1.069931341 \Rightarrow b = 0.067594479; note\,\lambda = e^{-\frac{a}{b}} \Rightarrow \log_e \lambda = -\frac{a}{b}$$

$$a = -0.067594479 \log 0.999078419 = -0.067594479 \times -0.0009220059368$$

$$a = 0.00006232251093; \mu_x = 0.00006232251093e^{0.067594479x}$$

4. DATA PRESENTATION AND ANALYSIS

Male Mortality Result from (R) Based On Gompertz Law

Table 1 Male Mortality Table based on Gompertz Model (R)

x	l_x	d_x	L_x	p_x	q_x	T_x	e_x	$\overset{0}{e}_x$	μ_x
20	98496	74	98459	0.999247	0.000753	8658446	88	88.5	0.000741
21	98422	76	98384	0.999224	0.000776	8559987	87	87.5	0.000764
22	98345	79	98306	0.9992	0.0008	8461604	86	86.5	0.000788
23	98267	81	98226	0.999175	0.000825	8363298	85	85.5	0.000813
24	98186	84	98144	0.999149	0.000851	8265071	84	84.5	0.000838
25	98102	86	98059	0.999123	0.000877	8166927	83	83.5	0.000864
26	98016	89	97972	0.999095	0.000905	8068868	82	82.5	0.000891
27	97927	91	97882	0.999067	0.000933	7970896	81	81.5	0.000919
28	97836	94	97789	0.999038	0.000962	7873015	81	81.5	0.000948
29	97742	97	97693	0.999008	0.000992	7775226	80	80.5	0.000977
30	97645	100	97595	0.998977	0.001023	7677532	79	79.5	0.001008
31	97545	103	97494	0.998945	0.001055	7579937	78	78.5	0.001039
32	97442	106	97389	0.998912	0.001088	7482443	77	77.5	0.001072
33	97336	109	97282	0.998879	0.001121	7385054	76	76.5	0.001105
34	97227	112	97171	0.998844	0.001156	7287773	75	75.5	0.001139
35	97115	116	97057	0.998808	0.001192	7190602	74	74.5	0.001175
36	96999	119	96939	0.99877	0.00123	7093545	73	73.5	0.001212
37	96880	123	96818	0.998732	0.001268	6996606	72	72.5	0.001249
38	96757	126	96694	0.998693	0.001307	6899787	71	71.5	0.001288
39	96630	130	96565	0.998652	0.001348	6803094	70	70.5	0.001328
40	96500	134	96433	0.99861	0.00139	6706529	70	70.5	0.00137
41	96366	138	96297	0.998567	0.001433	6610096	69	69.5	0.001413
42	96228	142	96157	0.998522	0.001478	6513799	68	68.5	0.001457
43	96085	146	96012	0.998476	0.001524	6417642	67	67.5	0.001502
44	95939	151	95864	0.998428	0.001572	6321630	66	66.5	0.001549
45	95788	155	95711	0.998379	0.001621	6225767	65	65.5	0.001597
46	95633	160	95553	0.998329	0.001671	6130056	64	64.5	0.001647
47	95473	165	95391	0.998277	0.001723	6034503	63	63.5	0.001698
48	95309	169	95224	0.998223	0.001777	5939112	62	62.5	0.001751
49	95139	174	95052	0.998168	0.001832	5843888	61	61.5	0.001806
50	94965	179	94875	0.998111	0.001889	5748835	61	61.5	0.001862
51	94786	185	94693	0.998052	0.001948	5653960	60	60.5	0.00192
52	94601	190	94506	0.997991	0.002009	5559267	59	59.5	0.00198
53	94411	196	94313	0.997929	0.002071	5464761	58	58.5	0.002042
54	94215	201	94115	0.997864	0.002136	5370447	57	57.5	0.002105
55	94014	207	93911	0.997798	0.002202	5276333	56	56.5	0.002171
56	93807	213	93701	0.997729	0.002271	5182422	55	55.5	0.002239
57	93594	219	93485	0.997659	0.002341	5088721	54	54.5	0.002308
58	93375	225	93262	0.997586	0.002414	4995236	54	54.5	0.00238
59	93150	232	93034	0.99751	0.00249	4901974	53	53.5	0.002455

60	92918	239	92798	0.997433	0.002567	4808940	52	52.5	0.002531
61	92679	245	92557	0.997353	0.002647	4716142	51	51.5	0.00261
62	92434	252	92308	0.997271	0.002729	4623585	50	50.5	0.002691
63	92182	259	92052	0.997186	0.002814	4531278	49	49.5	0.002775
64	91922	267	91789	0.997098	0.002902	4439226	48	48.5	0.002862
65	91655	274	91518	0.997008	0.002992	4347437	48	48.5	0.002951
66	91381	282	91240	0.996915	0.003085	4255919	47	47.5	0.003043
67	91099	290	90954	0.996819	0.003181	4164679	46	46.5	0.003138
68	90809	298	90660	0.99672	0.00328	4073724	45	45.5	0.003236
69	90511	306	90358	0.996617	0.003383	3983064	44	44.5	0.003337
70	90205	315	90048	0.996512	0.003488	3892705	43	43.5	0.003441
71	89891	323	89729	0.996404	0.003596	3802657	42	42.5	0.003548
72	89567	332	89401	0.996292	0.003708	3712928	42	42.5	0.003658
73	89235	341	89065	0.996176	0.003824	3623527	41	41.5	0.003772
74	88894	350	88719	0.996057	0.003943	3534462	40	40.5	0.00389
75	88544	360	88364	0.995935	0.004065	3445744	39	39.5	0.004011
76	88184	370	87999	0.995808	0.004192	3357380	38	38.5	0.004136
77	87814	380	87624	0.995678	0.004322	3269381	37	37.5	0.004265
78	87434	390	87240	0.995543	0.004457	3181757	36	36.5	0.004398
79	87045	400	86845	0.995405	0.004595	3094517	36	36.5	0.004535
80	86645	411	86440	0.995262	0.004738	3007672	35	35.5	0.004677
81	86234	421	86024	0.995115	0.004885	2921233	34	34.5	0.004823
82	85813	432	85597	0.994963	0.005037	2835209	33	33.5	0.004973
83	85381	443	85159	0.994806	0.005194	2749612	32	32.5	0.005128
84	84937	455	84710	0.994645	0.005355	2664453	31	31.5	0.005288
85	84482	467	84249	0.994478	0.005522	2579744	31	31.5	0.005453
86	84016	478	83777	0.994306	0.005694	2495494	30	30.5	0.005623
87	83538	490	83292	0.994129	0.005871	2411718	29	29.5	0.005798
88	83047	503	82796	0.993947	0.006053	2328425	28	28.5	0.005979
89	82544	515	82287	0.993759	0.006241	2245630	27	27.5	0.006165
90	82029	528	81765	0.993565	0.006435	2163343	26	26.5	0.006357
91	81501	541	81231	0.993365	0.006635	2081577	26	26.5	0.006555
92	80961	554	80684	0.993159	0.006841	2000346	25	25.5	0.00676
93	80407	567	80123	0.992946	0.007054	1919663	24	24.5	0.006971
94	79840	581	79549	0.992727	0.007273	1839540	23	23.5	0.007188
95	79259	594	78962	0.992501	0.007499	1759990	22	22.5	0.007412
96	78665	608	78361	0.992269	0.007731	1681029	21	21.5	0.007643
97	78056	622	77745	0.992028	0.007972	1602668	21	21.5	0.007881
98	77434	636	77116	0.991781	0.008219	1524923	20	20.5	0.008127
99	76798	651	76472	0.991526	0.008474	1447807	19	19.5	0.00838
100	76147	665	75814	0.991263	0.008737	1371334	18	18.5	0.008642
101	75482	680	75142	0.990992	0.009008	1295520	17	17.5	0.008911
102	74802	695	74454	0.990712	0.009288	1220378	16	16.5	0.009189
103	74107	710	73752	0.990424	0.009576	1145924	16	16.5	0.009475
104	73397	725	73035	0.990127	0.009873	1072172	15	15.5	0.009771

105	72673	740	72303	0.989821	0.010179	999137	14	14.5	0.010075
106	71933	755	71555	0.989505	0.010495	926834	13	13.5	0.010389
107	71178	770	70793	0.98918	0.01082	855278	12	12.5	0.010713
108	70408	785	70015	0.988844	0.011156	784486	11	11.5	0.011047
109	69622	801	69222	0.988499	0.011501	714470	10	10.5	0.011391
110	68822	816	68414	0.988142	0.011858	645248	9	9.5	0.011747
111	68006	831	67590	0.987775	0.012225	576835	9	9.5	0.012113
112	67174	847	66751	0.987396	0.012604	509245	8	8.5	0.01249
113	66328	862	65897	0.987006	0.012994	442494	7	7.5	0.01288
114	65466	877	65027	0.986603	0.013397	376598	6	6.5	0.013281
115	64589	892	64143	0.986189	0.013811	311570	5	5.5	0.013695
116	63697	907	63243	0.985761	0.014239	247428	4	4.5	0.014122
117	62790	922	62329	0.985321	0.014679	184185	3	3.5	0.014562
118	61868	936	61400	0.984866	0.015134	121856	2	2.5	0.015016
119	60932	951	60456	0.984398	0.015602	60456	1	1.5	0.015485
120	59981	59981	29990	0	1	29990	0	0.5	0.015967

The table 1 above shows the mortality results for male with initial 98496 survivors at age 20 and ending at age 120, the limit of life. The d_x in the above table increases as l_x decreases showing that as age increases more death are recorded. The p_x value ranges between 0 and 1 $(0 < p_x < 1)$. The q_x represents the probability of dying at different ages while p_x is the probability of survival at different ages. From table 1 above, when q_x increases, p_x decreases showing that as age increases the probability of dying tends to 1. When $q_x = 1, p_x = 0$ showing zero survival at limit of life. It was observed also that other functions of the mortality table decrease with age except the force of mortality μ_x which increases with age.

Female Mortality Result From (R) Based On Gormpertz Law

Table 2 Female Mortality Table based on Gompertz model (R)

x	l_x	d_x	L_x	p_x	q_x	T_x	e_x	$\overset{0}{e}_x$	μ_x
20	98957	25	98945	0.999747	0.000253	7421633	75	75.5	0.000241
21	98932	26	98919	0.999737	0.000263	7322688	74	74.5	0.000258
22	98906	28	98892	0.999717	0.000283	7223769	73	73.5	0.000276
23	98878	30	98863	0.999697	0.000303	7124877	72	72.5	0.000295
24	98848	33	98832	0.999666	0.000334	7026014	71	71.5	0.000316
25	98815	34	98798	0.999656	0.000344	6927183	70	70.5	0.000338
26	98781	37	98763	0.999625	0.000375	6828385	69	69.5	0.000361
27	98744	40	98724	0.999595	0.000405	6729622	68	68.5	0.000387
28	98704	42	98683	0.999574	0.000426	6630898	67	67.5	0.000414
29	98662	45	98640	0.999544	0.000456	6532215	66	66.5	0.000443
30	98617	48	98593	0.999513	0.000487	6433576	65	65.5	0.000474
31	98569	52	98543	0.999472	0.000528	6334983	64	64.5	0.000507
32	98517	55	98490	0.999442	0.000558	6236440	63	63.5	0.000542
33	98462	59	98433	0.999401	0.000599	6137950	62	62.5	0.00058
34	98403	63	98372	0.99936	0.00064	6039518	61	61.5	0.000621
35	98340	68	98306	0.999309	0.000691	5941146	60	60.5	0.000664
36	98272	72	98236	0.999267	0.000733	5842840	59	59.5	0.00071

37	98200	77	98162	0.999216	0.000784	5744-604	59	59.5	0.00076
38	98123	83	98082	0.999154	0.000846	5646443	58	58.5	0.000813
39	98040	88	97996	0.999102	0.000898	5548361	57	57.5	0.00087
40	97952	94	97905	0.99904	0.00096	5450365	56	56.5	0.000931
41	97858	101	97808	0.998968	0.001032	5352460	55	55.5	0.000996
42	97757	108	97703	0.998895	0.001105	5254653	54	54.5	0.001066
43	97649	115	97592	0.998822	0.001178	5156950	53	53.5	0.00114
44	97534	123	97473	0.998739	0.001261	5059358	52	52.5	0.00122
45	97411	131	97346	0.998655	0.001345	4961886	51	51.5	0.001305
46	97280	141	97210	0.998551	0.001449	4864540	50	50.5	0.001396
47	97139	150	97064	0.998456	0.001544	4767331	49	49.5	0.001494
48	96989	160	96909	0.99835	0.00165	4670267	48	48.5	0.001599
49	96829	171	96744	0.998234	0.001766	4573358	47	47.5	0.00171
50	96658	183	96567	0.998107	0.001893	4476614	46	46.5	0.00183
51	96475	195	96378	0.997979	0.002021	4380048	45	45.5	0.001958
52	96280	209	96176	0.997829	0.002171	4283670	45	45.5	0.002095
53	96071	222	95960	0.997689	0.002311	4187495	44	44.5	0.002241
54	95849	238	95730	0.997517	0.002483	4091535	43	43.5	0.002398
55	95611	253	95485	0.997354	0.002646	3995805	42	42.5	0.002566
56	95358	271	95223	0.997158	0.002842	3900320	41	41.5	0.002745
57	95087	288	94943	0.996971	0.003029	3805098	40	40.5	0.002937
58	94799	308	94645	0.996751	0.003249	3710155	39	39.5	0.003143
59	94491	328	94327	0.996529	0.003471	3615510	38	38.5	0.003362
60	94163	350	93988	0.996283	0.003717	3521183	37	37.5	0.003597
61	93813	373	93627	0.996024	0.003976	3427195	37	37.5	0.003849
62	93440	397	93242	0.995751	0.004249	3333568	36	36.5	0.004118
63	93043	423	92832	0.995454	0.004546	3240327	35	35.5	0.004406
64	92620	451	92395	0.995131	0.004869	3147495	34	34.5	0.004714
65	92169	479	91930	0.994803	0.005197	3055101	33	33.5	0.005044
66	91690	511	91435	0.994427	0.005573	2963171	32	32.5	0.005397
67	91179	543	90908	0.994045	0.005955	2871737	32	32.5	0.005774
68	90636	578	90347	0.993623	0.006377	2780829	31	31.5	0.006178
69	90058	613	89752	0.993193	0.006807	2690482	30	30.5	0.00661
70	89445	652	89119	0.992711	0.007289	2600731	29	29.5	0.007072
71	88793	693	88447	0.992195	0.007805	2511612	28	28.5	0.007567
72	88100	735	87733	0.991657	0.008343	2423165	28	28.5	0.008096
73	87365	779	86976	0.991083	0.008917	2335433	27	27.5	0.008662
74	86586	826	86173	0.99046	0.00954	2248457	26	26.5	0.009268
75	85760	876	85322	0.989785	0.010215	2162284	25	25.5	0.009916
76	84884	926	84421	0.989091	0.010909	2076962	25	25.5	0.010609
77	83958	980	83468	0.988327	0.011673	1992541	24	24.5	0.011351
78	82978	1036	82460	0.987515	0.012485	1909073	23	23.5	0.012145
79	81942	1095	81395	0.986637	0.013363	1826613	22	22.5	0.012995
80	80847	1154	80270	0.985726	0.014274	1745219	22	22.5	0.013903
81	79693	1217	79085	0.984729	0.015271	1664949	21	21.5	0.014876

82	78476	1282	77835	0.983664	0.016336	1585864	20	20.5	0.015916
83	77194	1348	76520	0.982538	0.017462	1508029	20	20.5	0.017029
84	75846	1416	75138	0.981331	0.018669	1431509	19	19.5	0.01822
85	74430	1486	73687	0.980035	0.019965	1356371	18	18.5	0.019494
86	72944	1557	72166	0.978655	0.021345	1282684	18	18.5	0.020857
87	71387	1630	70572	0.977167	0.022833	1210519	17	17.5	0.022316
88	69757	1702	68906	0.975601	0.024399	1139947	17	17.5	0.023876
89	68055	1775	67168	0.973918	0.026082	1071041	16	16.5	0.025546
90	66280	1848	65356	0.972118	0.027882	1003873	15	15.5	0.027332
91	64432	1920	63472	0.970201	0.029799	938517	15	15.5	0.029244
92	62512	1991	61517	0.96815	0.03185	875045	14	14.5	0.031289
93	60521	2060	59491	0.965962	0.034038	813529	14	14.5	0.033477
94	58461	2127	57398	0.963617	0.036383	754038	13	13.5	0.035818
95	56334	2190	55239	0.961125	0.038875	696640	13	13.5	0.038323
96	54144	2249	53020	0.958463	0.041537	641401	12	12.5	0.041003
97	51895	2302	50744	0.955641	0.044359	588382	12	12.5	0.04387
98	49593	2351	48418	0.952594	0.047406	537638	11	11.5	0.046938
99	47242	2392	46046	0.949367	0.050633	489220	11	11.5	0.05022
100	44850	2425	43638	0.945931	0.054069	443174	10	10.5	0.053732
101	42425	2450	41200	0.942251	0.057749	399537	10	10.5	0.05749
102	39975	2464	38743	0.938361	0.061639	358337	9	9.5	0.06151
103	37511	2469	36277	0.934179	0.065821	319594	9	9.5	0.065812
104	35042	2462	33811	0.929741	0.070259	283317	8	8.5	0.070414
105	32580	2443	31359	0.925015	0.074985	249506	8	8.5	0.075338
106	30137	2412	28931	0.919965	0.080035	218148	8	8.5	0.080607
107	27725	2366	26542	0.914662	0.085338	189217	7	7.5	0.086244
108	25359	2309	24205	0.908948	0.091052	162675	7	7.5	0.092275
109	23050	2238	21931	0.902907	0.097093	138470	6	6.5	0.098728
110	20812	2155	19735	0.896454	0.103546	116539	6	6.5	0.105632
111	18657	2059	17628	0.889639	0.110361	96805	5	5.5	0.113019
112	16598	1951	15623	0.882456	0.117544	79177	5	5.5	0.120922
113	14647	1835	13730	0.874718	0.125282	63555	5	5.5	0.129379
114	12812	1710	11957	0.866531	0.133469	49825	4	4.5	0.138426
115	11102	1577	10314	0.857954	0.142046	37868	4	4.5	0.148107
116	9525	1440	8805	0.848819	0.151181	27555	3	3.5	0.158464
117	8085	1301	7435	0.839085	0.160915	18750	3	3.5	0.169546
118	6784	1161	6204	0.828862	0.171138	11315	2	2.5	0.181402
119	5623	1023	5112	0.818069	0.181931	5112	1	1.5	0.194088
120	4600	4600	2300	0	1	2300	0	0.5	0.207661

Table 2 shows the female mortality experience for different ages. From the table, it is observed that the starting point for our study is 98957 at age 20 which gradually declines as age increases while the probability of dying at different ages increase. i.e. q_x increases with age. The probability of survival as well as curtate expectancy, future life expectancy and number of persons years lived at age x decreases as age increases. The number of death at exactly age x which is represented as d_x at the starting point was at the

increasing side to around age 50, then started dropping until it approaches the limit life while the force of mortality increased till the last age.

5. DISCUSSION OF RESULTS

The research sought to find out a way of estimating the parameters of Gompertz, obtain a model for the survival function and the force of mortality for both male and female. The gender group that experiences high rate of mortality and which ages are prone to high rate of mortality was also observed in the paper. It was observed that the male group has higher rate of mortality from our starting point but suddenly dropped at exactly age 49; i.e. age 20 to 49 with male having the probability of dying ranging from (0.000753 to 0.001832) while the female probability of dying ranges from (0.000253 to 0.001766). For higher ages above 49, a significant change in the mortality experience for both male and female was observed. It was observed that from age 50 to 120, female has higher probability of dying; for $0 < 001893 \leq 1$ while the male probability ranges from 0.001889 to 1 for $0 < 0.001889 \leq 1$.

Also, it was observed, female age group 93 to 109 experienced the highest number of deaths (2060-2238) and started declining from age 110 till it gets to 120.

The number of deaths for male was at the increasing side from the beginning of the study till it gets to age 119 having 951 deaths recorded before the limit of life.

6. CONCLUSION

Modeling mortality experience for high ages has been a subject of concern for insurance practitioners and actuarial scientist. The focus of this paper is to assess a way of modeling mortality by applying Gompertz model in order to ease out financial burden of death. The mortality table has summarized the mortality experience of a particular group by showing at each age the probability of dying or surviving, future life expectancy and number of lives surviving particular age, number of deaths and the force of mortality intensity. This paper mainly appraises law of mortality and preparation of mortality table based on Gompertz model. From the mortality table, the life expectancy, representing the mean of the life table distribution of deaths, is the indicator often used to describe this distribution. In a defined population experiencing high level of infant mortality conditions, life expectancy will be within the age range of premature deaths, even when most deaths occur around ages zero and the modal age at death.

Competing Interests

All authors have declared that there is no competing interests.

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Authors' contributions

This work was carried out in collaboration between all authors. Author OGM designed, analyzed, interpreted and prepared the first draft of the manuscript. Author OSK managed the literature searches, analyses and prepared the final draft of the manuscript. All authors read and approved the final draft of the manuscript.

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